1.) (a) $\det \boldsymbol{A} = 2$ (A1)

$$\boldsymbol{A}^{-1} = \frac{1}{2}\begin{pmatrix}3 & 4\\1 & 2\end{pmatrix} \left(= \begin{pmatrix}\frac{3}{2} & 2\\ \frac{1}{2} & 1\end{pmatrix}\right)$$ A1 N22

(b) evidence of multiplying by $\boldsymbol{A}^{-1}$ (M1)

e.g. $\boldsymbol{X} = \boldsymbol{A}^{-1}\begin{pmatrix}4 & 6\\2 & -2\end{pmatrix}, \boldsymbol{A}^{-1}\boldsymbol{B}$

correct working A1

e.g. $\boldsymbol{X} = \frac{1}{2}\begin{pmatrix}3 & 4\\1 & 2\end{pmatrix}\begin{pmatrix}4 & 6\\2 & -2\end{pmatrix}, \begin{pmatrix}\frac{3}{2} & 2\\ \frac{1}{2} & 1\end{pmatrix}\begin{pmatrix}4 & 6\\2 & -2\end{pmatrix}, \frac{1}{2}\begin{pmatrix}20 & 10\\8 & 2\end{pmatrix}$

$$\boldsymbol{X} = \begin{pmatrix}10 & 5\\4 & 1\end{pmatrix}$$ A2 N34

[6]

2.) (a) evidence of considering determinant (M1)

e.g. $3 \times -3 - (-2) \times x$, attempt to find inverse

setting the determinant equal to zero (M1)

e.g. $-9 + 2x = 0$, $2x = 9$

$$x = \frac{9}{2}$$ A1 N23

(b) **METHOD 1**

$$\boldsymbol{A}^{-1} = \frac{1}{-9+2x}\begin{pmatrix}-3 & -x\\2 & 3\end{pmatrix}$$ (A1) (A1)

Note: *Award A1 for* $\frac{1}{\det \boldsymbol{A}}$, *A1 for* $\begin{pmatrix}-3 & -x\\2 & 3\end{pmatrix}$.

one correct equation from $\boldsymbol{A} = \boldsymbol{A}^{-1}$ (A1)

e.g. $\frac{-3}{-9+2x} = 3, \frac{2}{-9+2x} = 2, \frac{3}{-9+2x} = -3, \frac{-x}{-9+2x} = x$

attempt to solve the equation (M1)

e.g. $-3 = 3(-9 + 2x)$, $-9 + 2x = -1$

$x = 4$ (do **not** accept $x = 4$, $x = 0$) A1 N45

METHOD 2

$\boldsymbol{A}^2 = \boldsymbol{I}$ (A1)

$$\boldsymbol{A}^2 = \begin{pmatrix}9-2x & 0\\0 & -2x+9\end{pmatrix}$$ (A1)

one correct equation from $\boldsymbol{A}^2 = \boldsymbol{I}$ (A1)

e.g. $9 - 2x = 1$

attempt to solve the equation (M1)

e.g. $2x = 8$

$x = 4$ A1 N45

[8]

3.) (a) $\boldsymbol{M} = \begin{pmatrix} 1 & 6 & -3 \\ 4 & 2 & -4 \\ 1 & 1 & 5 \end{pmatrix}, \boldsymbol{N} = \begin{pmatrix} -1 \\ 12 \\ 15 \end{pmatrix}$ A2A1 N3 3

(b) evidence of appropriate approach (M2)

e.g. $\boldsymbol{X} = \boldsymbol{M}^{-1}\boldsymbol{N}$, attempting to solve a system of three equations

$\boldsymbol{X} = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$ A1 N33

(c) $x = 5, y = 0, z = 2$ A1 N11

[7]

4.) (a) evidence of multiplying (M1)

e.g. one correct element

$\boldsymbol{AB} = \begin{pmatrix} -15 \\ 5 \end{pmatrix}$ A1A1 N3

(b) **METHOD 1**

evidence of multiplying by $\boldsymbol{A}$ (on left or right) (M1)

e.g. $\boldsymbol{AA}^{-1}\boldsymbol{X} = \boldsymbol{AB}$, $\boldsymbol{X} = \boldsymbol{AB}$

$\boldsymbol{X} = \begin{pmatrix} -15 \\ 5 \end{pmatrix}$ (accept $x = -15, y = 5$) A1N2

METHOD 2

attempt to set up a system of equations (M1)

e.g. $\frac{4x+2y}{10} = -5, \frac{-3x+y}{10} = 5$

$\boldsymbol{X} = \begin{pmatrix} -15 \\ 5 \end{pmatrix}$ (accept $x = -15, y = 5$) A1N2

[5]

5.) (a) $\boldsymbol{A}^{-1} = \begin{pmatrix} -4.33 & -2 & 1.67 \\ 1.67 & 1 & -0.333 \\ -0.667 & 0 & 0.333 \end{pmatrix}$ $\left(= \begin{pmatrix} -\frac{13}{3} & -2 & \frac{5}{3} \\ \frac{5}{3} & 1 & -\frac{1}{3} \\ -\frac{2}{3} & 0 & \frac{1}{3} \end{pmatrix} \right)$ A2 N2

(b) evidence of attempting to solve equation (M1)
e.g. multiply by $\boldsymbol{A}^{-1}$ (on left or right), setting up system of equations

$\boldsymbol{X} = \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix}$ (accept $x = 1$, $y = 0$, $z = -1$) A2N3

[5]

6.) (a) correct substitution into the formula for the determinant (A1)
e.g. $\det \boldsymbol{A} = 9e^x \times e^{3x} - e^x \times e^x$

$\det \boldsymbol{A} = 9e^{4x} - e^{2x}$ A1N2

(b) recognizing that no inverse implies $\det \boldsymbol{A} = 0$ R1
e.g. $9e^{4x} - e^{2x} = 0$, $ad - bc = 0$

attempt to solve equation (M1)
e.g. $e^{2x} = \frac{1}{9}$, $e^{-2x} = 9$, $e^{2x}(9e^{2x} - 1) = 0$, $9e^{4x} = e^{2x}$

rearranging to get correct log equation
e.g. $2x = \ln\frac{1}{9}, -2x = \ln 9, \ln(9e^{4x}) = \ln(e^{2x})$ (A1)

isolating x A1
e.g. $x\ \frac{1}{2}\ln\frac{1}{9}, x = -\frac{1}{2}\ln 9, x = \ln\frac{1}{3},\ a = -\frac{1}{2}, b = 9$

$x = -\ln 3$ (accept $a = -1$, $b = 3$) A1N3

[7]

7.) (a) (i) $\boldsymbol{AB} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$ $(= 4\boldsymbol{I})$ A2 N2

(ii) $\boldsymbol{A}^{-1} = \frac{1}{4}\begin{pmatrix} 2 & -1 \\ -6 & 5 \end{pmatrix}, \frac{1}{4}\boldsymbol{B}, \begin{pmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{3}{2} & \frac{5}{4} \end{pmatrix}$ A1N1

(b) **METHOD 1**

$\begin{pmatrix} x \\ y \end{pmatrix} = \boldsymbol{A}^{-1}\boldsymbol{C}$ (M1)

$= \frac{1}{4}\begin{pmatrix} 2 & -1 \\ -6 & 5 \end{pmatrix}\begin{pmatrix} 8 \\ -4 \end{pmatrix}$ $\left(\begin{pmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{3}{2} & \frac{5}{4} \end{pmatrix}\begin{pmatrix} 8 \\ -4 \end{pmatrix} \right)$ A1

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -17 \end{pmatrix}$ A1A1N3

METHOD 2

$5x + y = 8$, $6x + 2y = -4$ A1

for work towards solving **their** system (M1)

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -17 \end{pmatrix}$ A1A1N3

[7]

8.) (a) **METHOD 1**

$\boldsymbol{M} = (\boldsymbol{M}^{-1})^{-1}$ (M1)

$\boldsymbol{M} = \frac{1}{10}\begin{pmatrix} 2 & 0 \\ -1 & 5 \end{pmatrix}$ A1A1N3

METHOD 2

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 5 & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (M1)

$5a + b = 1$, $2b = 0$, $5c + d = 0$, $2d = 1$ (A1)

$\boldsymbol{M} = \begin{pmatrix} 0.2 & 0 \\ -0.1 & 0.5 \end{pmatrix}$ A1N3

(b) **METHOD 1**

evidence of appropriate approach (M1)

e.g. $\boldsymbol{X} = \boldsymbol{M}^{-1}\boldsymbol{B}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 7 \end{pmatrix}$ A1

$= \begin{pmatrix} 5 \\ 15 \end{pmatrix}$ A1N2

METHOD 2

evidence of appropriate approach (M1)

e.g. $\begin{pmatrix} 0.2 & 0 \\ -0.1 & 0.5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$

$0.2x = 1$, $-0.1x + 0.5y = 7$ A1

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 15 \end{pmatrix}$ A1N2

[6]

9.) (a) evidence of substituting $(-4, 3)$ (M1)

correct substitution $3 = a(-4)^2 + b(-4) + c$ A1

$16a - 4b + c = 3$ AG N0

(b) $3 = 36a + 6b + c$, $-1 = 4a - 2b + c$ A1A1N1N1

(c) (i) $\boldsymbol{A} = \begin{pmatrix} 16 & -4 & 1 \\ 36 & 6 & 1 \\ 4 & -2 & 1 \end{pmatrix}; \boldsymbol{B} = \begin{pmatrix} 3 \\ 3 \\ -1 \end{pmatrix}$ A1A1 N1N1

(ii) $\boldsymbol{A}^{-1} = \begin{pmatrix} 0.05 & 0.0125 & -0.0625 \\ -0.2 & 0.075 & 0.125 \\ -0.6 & 0.1 & 1.5 \end{pmatrix} \quad \left(= \begin{pmatrix} \frac{1}{20} & \frac{1}{80} & -\frac{1}{16} \\ -\frac{1}{5} & \frac{3}{40} & \frac{1}{8} \\ -\frac{3}{5} & \frac{1}{10} & \frac{3}{2} \end{pmatrix} \right)$ A2N2

(iii) evidence of appropriate method (M1)

e.g. $\boldsymbol{X} = \boldsymbol{A}^{-1}\boldsymbol{B}$, attempting to solve a system of three equations

$\boldsymbol{X} = \begin{pmatrix} 0.25 \\ -0.5 \\ -3 \end{pmatrix}$ (accept fractions) A2

$f(x) = 0.25x^2 - 0.5x - 3$ (accept $a = 0.25$, $b = -0.5$, $c = -3$, or fractions) A1N2

(d) $f(x) = 0.25(x - 1)^2 - 3.25$ (accept $h = 1$, $k = -3.25$, $a = 0.25$, or fractions) A1A1A1N3

[15]

10.) (a) $\boldsymbol{A}^{-1} = \begin{pmatrix} 3 & 2 & -3 \\ 2 & 1 & -2 \\ -8 & -6 & 9 \end{pmatrix}$ A2 N2

(b) evidence of subtracting matrices (M1)

e.g. $\begin{pmatrix} 7 & 6 & -7 \\ 6 & 5 & -8 \\ 1 & 7 & -5 \end{pmatrix} - \begin{pmatrix} -3 & 2 & 1 \\ 5 & 3 & 4 \\ -9 & 2 & 10 \end{pmatrix}, \begin{pmatrix} 10 & 4 & -8 \\ 1 & 2 & -12 \\ 10 & 5 & -15 \end{pmatrix}, \boldsymbol{D} - \boldsymbol{C}$

evidence of multiplying **on left** by $\boldsymbol{A}^{-1}$ (M1)

e.g. $\boldsymbol{A}^{-1}\boldsymbol{AB}, \boldsymbol{A}^{-1}(\boldsymbol{D} - \boldsymbol{C}), \begin{pmatrix} 3 & 2 & -3 \\ 2 & 1 & -2 \\ -8 & -6 & 9 \end{pmatrix}\begin{pmatrix} 10 & 4 & -8 \\ 1 & 2 & -12 \\ 10 & 5 & -15 \end{pmatrix}$

$\boldsymbol{B} = \begin{pmatrix} 2 & 1 & -3 \\ 1 & 0 & 2 \\ 4 & 1 & 1 \end{pmatrix}$ A2N3

[6]

11.) (a) $\det \boldsymbol{M} = -4$ A1 N1

(b) $\boldsymbol{M}^{-1} = -\frac{1}{4}\begin{pmatrix} -1 & -1 \\ -2 & 2 \end{pmatrix} \quad \left(= \begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \right)$ A1A1 N2

Note: *Award A1 for* $-\frac{1}{4}$ *and A1 for the correct matrix.*

(c) $\boldsymbol{X} = \boldsymbol{M}^{-1}\begin{pmatrix}4\\8\end{pmatrix}$ $\left(\boldsymbol{X} = -\frac{1}{4}\begin{pmatrix}-1 & -1\\-2 & 2\end{pmatrix}\begin{pmatrix}4\\8\end{pmatrix}\right)$ M1

$\boldsymbol{X} = \begin{pmatrix}3\\-2\end{pmatrix}$ $(x=3, y=-2)$ A1A1 N0

Note: *Award no marks for an **algebraic** solution of the system 2x + y = 4, 2x - y = 8.*

[6]

12.) (a) evidence of correct method (M1)
e.g. at least 1 correct element (must be in a 2 × 2 matrix)

$\boldsymbol{AB} = \begin{pmatrix}-2-2q & 0\\-6+pq & 3+\frac{p}{2}\end{pmatrix}$ A1 N2

(b) **METHOD 1**

evidence of using $\boldsymbol{AB} = \boldsymbol{I}$ (M1)
2 correct equations A1A1

e.g. $-2 - 2q = 1$ and $3 + \frac{p}{2} = 1$, $-6 + pq = 0$

$p = -4$, $q = -\frac{3}{2}$ A1A1 N1N1

METHOD 2

finding $\boldsymbol{A}^{-1} = \frac{1}{p+6}\begin{pmatrix}p & 2\\-3 & 1\end{pmatrix}$ A1

evidence of using $\boldsymbol{A}^{-1} = \boldsymbol{B}$ (M2)

e.g. $\frac{2}{p+6} = 1$ and $-\frac{3}{p+6} = q$, $\frac{p}{p+6} = -2$ and $-\frac{3}{p+6} = q$

$p = -4$, $q = -\frac{3}{2}$ A1A1 N1N1

[7]

13.) (a) $\boldsymbol{A}^{-1} = \begin{pmatrix}-1 & 1 & -1\\1 & -0.5 & 1.25\\1 & -0.5 & 0.75\end{pmatrix}$ A2 N2

(b) (i) $\boldsymbol{I} - \frac{1}{2}\boldsymbol{B} = \boldsymbol{A}^{-1}$ A1

$-\frac{1}{2}\boldsymbol{B} = \boldsymbol{A}^{-1} - \boldsymbol{I}$ A1

$\boldsymbol{B} = -2(\boldsymbol{A}^{-1} - \boldsymbol{I})$ AG

(ii) $\boldsymbol{B} = \begin{pmatrix}4 & -2 & 2\\-2 & 3 & -2.5\\-2 & 1 & 0.5\end{pmatrix}$ A2 N2

(iii) $\det \boldsymbol{B} = 12$ A1 N1

(iv) det $\boldsymbol{B} \neq 0$ R1 N1

(c) (i) evidence of using a valid approach M1

e.g. $\boldsymbol{X} = \boldsymbol{B}^{-1}\boldsymbol{C}$

$$\boldsymbol{X} = \begin{pmatrix} 0.333 \\ 1 \\ 1.33 \end{pmatrix} \quad \left(= \begin{pmatrix} \frac{1}{3} \\ 1 \\ \frac{4}{3} \end{pmatrix} \right)$$ A1 N1

(ii) $4x - 2y + 2z = 2$, $-2x + 3y - 2.5z = -1$, $-2x + y + 0.5z = 1$ A1A1A1 N3

[13]

14.) (a) det $\boldsymbol{A} = 5$ (A1)

$$\boldsymbol{A}^{-1} = \frac{1}{5}\begin{pmatrix} 3 & -8 \\ -2 & 7 \end{pmatrix}$$ A1 N2

(b) Set up matrix equation $\begin{pmatrix} 7 & 8 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ (M1)

premultiplying by $\boldsymbol{A}^{-1}$ M1

$$\boldsymbol{A}^{-1}\begin{pmatrix} 7 & 8 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \boldsymbol{A}^{-1}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{5}\begin{pmatrix} -5 \\ 5 \end{pmatrix} \quad \left(\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right)$$ A1

$x = -1$, $y = 1$ A1N0

[6]

15.) (a)

$$\boldsymbol{A}^{-1} = \begin{pmatrix} 0.1 & 0.4 & 0.1 \\ -0.7 & 0.2 & 0.3 \\ -1.2 & 0.2 & 0.8 \end{pmatrix}$$ A2 N2

(b) For recognizing that the equations may be written as $\boldsymbol{A}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ (M1)

for attempting to calculate $\begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{pmatrix} = \boldsymbol{A}^{-1}\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad \left(= \begin{pmatrix} 1.2 \\ 0.6 \\ 1.6 \end{pmatrix} \right)$ M1

$x = 1.2$, $y = 0.6$, $z = 1.6$ (accept row or column vectors) A2N3

[6]

16.) (a)

$$\boldsymbol{A}^{-1} = \begin{pmatrix} -0.2 & 1.8 & -0.6 \\ -0.4 & 0.6 & -0.2 \\ 0.4 & -2.6 & 1.2 \end{pmatrix}$$ A2 N2

(b) For recognizing that the equations may be written as $\boldsymbol{A}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ (M1)

For attempting to calculate $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathrm{A}^{-1}\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ (M1)

$x = 4$, $y = 1$, $z = -6$ A2 N4

[6]

17.) (a) (i) $\boldsymbol{A}^{-1} = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}$ A2 N2

(ii) $\boldsymbol{A}^2 = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$ A2 N2

(b) $\begin{pmatrix} 0 & 4 \\ 4 & 0 \end{pmatrix} + \begin{pmatrix} p & 2 \\ 0 & q \end{pmatrix} = \begin{pmatrix} 2 & 6 \\ 4 & 3 \end{pmatrix}$ (M1)

$p = 2$, $q = 3$ A1A1 N3

(c) Evidence of attempt to multiply (M1)

eg $\boldsymbol{A}^{-1}\boldsymbol{B} = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}\begin{pmatrix} 2 & 2 \\ 0 & 3 \end{pmatrix}$

$\boldsymbol{A}^{-1}\boldsymbol{B} = \begin{pmatrix} 0 & \frac{3}{2} \\ 1 & 1 \end{pmatrix}$ $\left(\text{accept} \begin{pmatrix} 0 & \frac{1}{2}q \\ \frac{1}{2}p & 1 \end{pmatrix}\right)$ A1 N2

(d) Evidence of correct approach (M1)

eg $\boldsymbol{X} = \boldsymbol{A}^{-1}\boldsymbol{B}$, setting up a system of equations

$\boldsymbol{X} = \begin{pmatrix} 0 & \frac{3}{2} \\ 1 & 1 \end{pmatrix}$ $\left(\text{accept} \begin{pmatrix} 0 & \frac{1}{2}q \\ \frac{1}{2}p & 1 \end{pmatrix}\right)$ A1 N2

[11]

18.) (a) $\boldsymbol{A}^{-1} = \begin{pmatrix} -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{5}{3} & -\frac{7}{3} \\ \frac{2}{3} & -\frac{4}{3} & \frac{5}{3} \end{pmatrix}$ or $\begin{pmatrix} -0.333 & 0.667 & -0.333 \\ -0.333 & 1.67 & -2.33 \\ 0.667 & -1.33 & 1.67 \end{pmatrix}$ A2 N2

(b) (i) $\boldsymbol{X} = \boldsymbol{A}^{-1}\boldsymbol{B}$ A1 N1

(ii) $\boldsymbol{X} = \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}$ A3 N3

[6]

19.) (a) (i) $a = 4$ A1 N1

(ii) $b = 7$ A1 N1

(b) **EITHER**

$\boldsymbol{A}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 10 \\ -12 \end{pmatrix}$ A1 N1

OR

$\begin{pmatrix} 1 & 2 & 0 \\ -3 & 1 & -1 \\ 2 & -2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 10 \\ -12 \end{pmatrix}$ A1 N1

(c) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \boldsymbol{A}^{-1}\begin{pmatrix} 7 \\ 10 \\ -12 \end{pmatrix}$ (accept algebraic method) (M1)

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 5 \\ 4 \end{pmatrix}$ (accept $x = -3$, $y = 5$, $z = 4$) A2 N3

[6]

20.) (a) $\begin{pmatrix} 0.1 & 0.4 & 0.1 \\ -0.7 & 0.2 & 0.3 \\ -1.2 & 0.2 & 0.8 \end{pmatrix}$ A2 3

(b) For recognizing that the equations may be written as A $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ (M1)

for attempting to calculate $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ $\left(= \begin{pmatrix} 1.2 \\ 0.6 \\ 1.6 \end{pmatrix} \right)$ M1

$x = 1.2$, $y = 0.6$, $z = 1.6$ (Accept row or column vectors) A2 3

[6]

21.) (a) $3\boldsymbol{Q} = \begin{pmatrix} -4 & 8 \\ 2 & 14 \end{pmatrix} + \begin{pmatrix} 5 & 2 \\ -1 & a \end{pmatrix}$ (A1)

$$3\boldsymbol{Q} = \begin{pmatrix} -9 & 6 \\ 3 & 14-a \end{pmatrix} \qquad \text{(A1)}$$

$$\boldsymbol{Q} = \begin{pmatrix} -3 & 2 \\ 1 & \frac{14-a}{3} \end{pmatrix} \qquad \text{(A1)} \quad \text{(N3)} \quad 3$$

(b) $\boldsymbol{CD} = \begin{pmatrix} -2 & 4 \\ 1 & 7 \end{pmatrix}\begin{pmatrix} 5 & 2 \\ -1 & a \end{pmatrix}$

$= \begin{pmatrix} -14 & -4+4a \\ -2 & 2+7a \end{pmatrix}$ (A1)(A1)(A1)(A1) (N4)4

(c) $\det \boldsymbol{D} = 5a+2$ (may be implied) (A1)

$\boldsymbol{D}^{-1} = \frac{1}{5a+2}\begin{pmatrix} a & -2 \\ 1 & 5 \end{pmatrix}$ (A1) (N2)2

[9]

22.) (a) $\boldsymbol{A}^{-1} = \begin{pmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{7}{3} & \frac{5}{3} \end{pmatrix}$ or $\frac{1}{3}\begin{pmatrix} 2 & -1 \\ -7 & 5 \end{pmatrix}$ or $\begin{pmatrix} 0.667 & -0.333 \\ -2.33 & -1.67 \end{pmatrix}$ (A1)(A1) (N2)

(b) $\boldsymbol{AX} = \boldsymbol{C} - \boldsymbol{B}$ (may be implied) (A1)

$\boldsymbol{X} = \boldsymbol{A}^{-1}(\boldsymbol{C}-\boldsymbol{B})$ (A1)

$\boldsymbol{D} = \boldsymbol{C} - \boldsymbol{B}$

$= \begin{pmatrix} 7 & -11 \\ 11 & -13 \end{pmatrix}$ (A1) (N3)

(c) $\boldsymbol{X} = \begin{pmatrix} 1 & -3 \\ 2 & 4 \end{pmatrix}$ (A2) (N2)

[7]

23.) (a) $\det \boldsymbol{A} = 5(1) - 7(-2) = 19$

$$\boldsymbol{A}^{-1} = \frac{1}{19}\begin{pmatrix} 1 & 2 \\ -7 & 5 \end{pmatrix} = \begin{pmatrix} \frac{1}{19} & \frac{2}{19} \\ \frac{-7}{19} & \frac{5}{19} \end{pmatrix} \qquad \text{(A2)}$$

***Note:** Award (A1) for* $\begin{pmatrix} 1 & 2 \\ -7 & 5 \end{pmatrix}$, *(A1) for dividing by 19.*

OR

$$\boldsymbol{A}^{-1} = \begin{pmatrix} 0.0526 & 0.105 \\ -0.368 & 0.263 \end{pmatrix}$$ (G2) 2

(b) (i) $\boldsymbol{XA} + \boldsymbol{B} = \boldsymbol{C} \Rightarrow \boldsymbol{XA} = \boldsymbol{C} - \boldsymbol{B}$ (M1)

$\boldsymbol{X} = (\boldsymbol{C} - \boldsymbol{B})\boldsymbol{A}^{-1}$ (A1)

OR

$\boldsymbol{X} = (\boldsymbol{C} - \boldsymbol{B})\boldsymbol{A}^{-1}$ (A2)

(ii) $$(\boldsymbol{C} - \boldsymbol{B})\boldsymbol{A}^{-1} = \begin{pmatrix} -11 & -7 \\ -13 & 9 \end{pmatrix}\begin{pmatrix} \frac{1}{19} & \frac{2}{19} \\ \frac{-7}{19} & \frac{5}{19} \end{pmatrix}$$ (A1)

$$\Rightarrow \boldsymbol{X} = \begin{pmatrix} \frac{38}{19} & \frac{-57}{19} \\ \frac{-76}{19} & \frac{19}{19} \end{pmatrix} = \begin{pmatrix} 2 & -3 \\ -4 & 1 \end{pmatrix}$$ (A1)

OR

$$\boldsymbol{X} = \begin{pmatrix} 2 & -3 \\ -4 & 1 \end{pmatrix}$$ (G2) 4

Note: *If premultiplication by* $\boldsymbol{A}^{-1}$ *is used, award (M1)(M0) in part (i) but award (A2) for* $\begin{pmatrix} \frac{-37}{19} & \frac{11}{19} \\ \frac{12}{19} & \frac{94}{19} \end{pmatrix}$ *in part (ii).*

[6]

24.) (a) $$M^2 = \begin{pmatrix} a & 2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} a & 2 \\ 2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} a^2 + 4 & 2a - 2 \\ 2a - 2 & 5 \end{pmatrix}$$ (A1)(A1)(A1)(A1) 4

(b) $2a - 2 = -4$

$\Rightarrow a = -1$ (A1)

Substituting: $a^2 + 4 = (-1)^2 + 4 = 5$ (A1) 2

Note: *Candidates may solve* $a^2 + 4 = 5$ *to give* $a = \pm 1$*, and then show that only* $a = -1$ *satisfies* $2a - 2 = -4$*.*

(c) $$M = \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}$$

$$M^{-1} = -\frac{1}{3}\begin{pmatrix} -1 & -2 \\ -2 & -1 \end{pmatrix}$$ (M1)

$$= \frac{1}{3}\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \text{ or } \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix}$$ (A1)

$-x + 2y = -3$

$2x - y = 3$

$$\Rightarrow \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \end{pmatrix}$$ (M1)(M1)

$$\Rightarrow \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} -3 \\ 3 \end{pmatrix}$$ (A1)

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$ (A1) 6

ie $x = 1$

$y = -1$

Note: *The solution must use matrices. Award no marks for solutions using other methods.*

[12]

25.) $B = (BA)A^{-1}$ (M1)

$= -\frac{1}{4}\begin{pmatrix} 11 & 2 \\ 44 & 8 \end{pmatrix}\begin{pmatrix} 0 & -2 \\ -2 & 5 \end{pmatrix}$ (M1)

$= -\frac{1}{4}\begin{pmatrix} -4 & -12 \\ -16 & -48 \end{pmatrix}$ (A1)

$= \begin{pmatrix} 1 & 3 \\ 4 & 12 \end{pmatrix}$ (A1)

OR

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 5 & 2 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 11 & 2 \\ 44 & 8 \end{pmatrix}$ (M1)

$\Rightarrow \left.\begin{aligned} 5a + 2b &= 11 \\ 2a &= 2 \end{aligned}\right\}$

$\Rightarrow a = 1, b = 3$ (A1)

$\left.\begin{aligned} 5c + 2d &= 44 \\ 2c &= 8 \end{aligned}\right\}$

$\Rightarrow c = 4, d = 12$ (A1)

$B = \begin{pmatrix} 1 & 3 \\ 4 & 12 \end{pmatrix}$ (A1) (C4)

Note: *Correct solution with inversion (ie AB instead of BA) earns FT marks, (maximum [3 marks]).*

[4]